Laser drilling in materials

Laser drilling is a new technique which is used for producing high-aspect ratio holes.

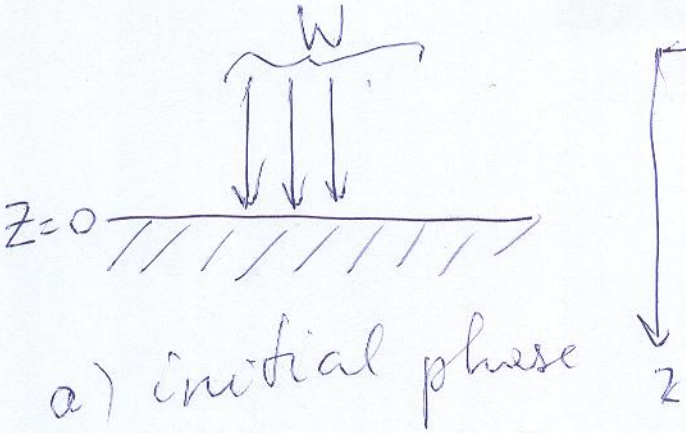
Laser drilling occurs through

- melting
- vaporization (ablation)

of the material through absorption of energy from a focused laser beam.

Whether melting or vaporization is more dominant depends on many factors, and laser pulse duration and energy are playing most important roles

Schematic description of this process (laser drilling)

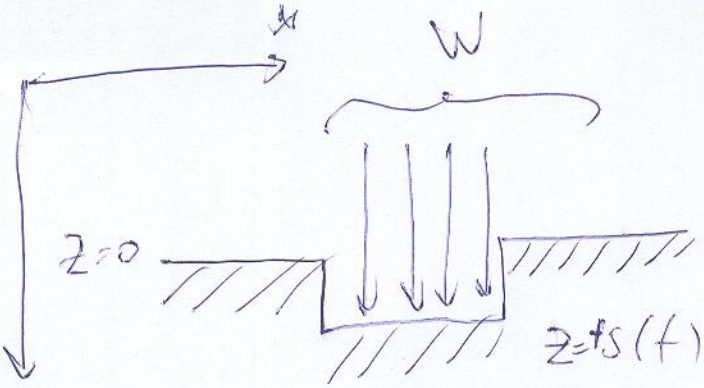


a) initial phase

W - laser pulse power on the order $10 \div 100$ $\frac{MWatt}{cm^2}$

The rate at which energy is used

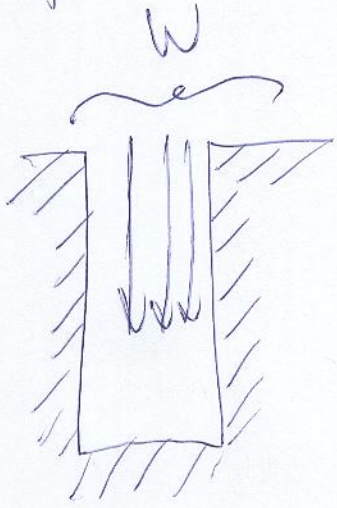
Temperature of the surface increases



b) the front $s(t)$ starts to move

$1W = 1\frac{J}{s}$

$\frac{ds}{dt} < v$ and the velocity increases



c

$\frac{ds}{dt} \to v$

The front moves with constant maximum velocity

a) a part of laser beam energy is absorbed at the surface

b) the temperature $T(t)$ of the material near a surface starts to grow

c) When this temperature reaches a critical value (boiling temperature) the heat (energy) is still absorbed but the temperature remains not changed and liquid material transforms into gas form (state) and the metal evaporate.

d) We will not consider the reflection of laser beam

e) We will not consider movement of the liquid formed during the experiment

The obtained mathematical model will give a possibility to get an approximation of the exact processes

We assume that energy W is distributed uniformly in the domain and the area of this domain is equal to A.

• The amount of heat generated during time Δt is $W \cdot \Delta t$.

During this time we evaporate a metal layer of thickness Δs, the volume of the column is $\Delta V = A \cdot \Delta s$.

Law of Conservation of Energy

$$W \Delta t = h \rho A \Delta s$$

ρ - a density of metal,

h - amount of heat required to evaporate one unit of mass of a substance.

Lets take a limit when $\Delta t \to 0$ and get differential equation

$$\frac{ds}{dt} = \frac{W/A}{h\rho}, \quad s(0) = 0.$$

Analysis of dimensions

(prove the validity of MM)

$$\left[\frac{ds}{dt}\right] = \frac{m}{s}$$

$$\boxed{\frac{dS}{dt} = \frac{W/A}{h\,\rho}}$$

$$[\rho] = \frac{kg}{m^3}$$

$$[A] = m^2$$

$$[h] = \frac{J}{kg}$$

$$[W] = J/s \rightarrow \text{Watt } (W)$$

$$\left[\frac{W/A}{h\,\rho}\right] = \frac{J}{s \cdot m^2} \times \frac{kg}{J} \times \frac{m^3}{kg} = \frac{m}{s}$$

It follows from our MM, that the velocity of the front $\frac{ds}{dt}$ is constant

$$\frac{ds}{dt} := v = \frac{W/A}{h\,\rho}$$

and it describes only the third stage c when the dynamics of the front reaches the maximum velocity.

If it is important to simulate more accurately (precisely) the initial stage of the process we should modify the mathematical model.

How to compute coefficient h.

1. First the surface should be heated till temperature

T_e - melting point

h_1 - specific heat capacity

We need to use the following amount of energy

$$(T_e - T_0)\, h_1 .$$

2. The information on heating process

h_2 - specific heat of fussion (lydimosi spec. šilumà)

h_3 - specific heat of vaporization (garavimo specifinė šilumà)

$$h = (T_e - T_0)\, h_1 + h_2 + h_3$$

Now we can integrate the given differential equation and to find dynamics of $s(t)$:

$$s(t) = \frac{1}{h\rho A} \int_0^t W(t')\, dt'$$

(here $W = W(t)$)

$$= \frac{E(t)}{h\rho A}$$

If $W(t) = W_0$ (constant laser beam power)

$$\boxed{s(t) = \frac{W_0 t}{h\rho A}}$$

The height of column $s(t)$ depends only on a full amount of energy $E(t)$.

In Table 1 we present typical values of h components for different metals

Table 1. $[h] = kJ/kg$

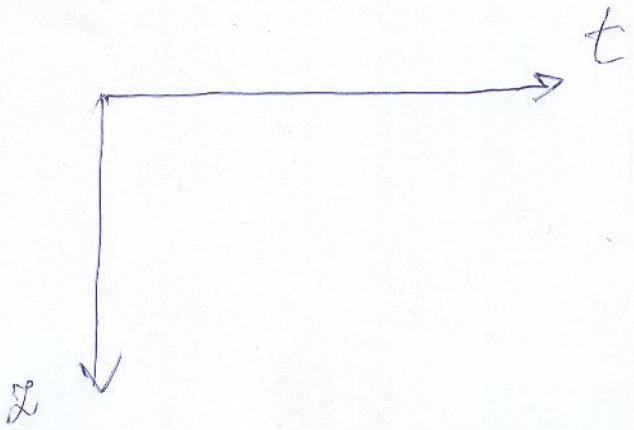
Metal	$h_1(T_e - T_0)$	h_2	h_3
Aluminium	2490	389	10800
Gold	368	66,9	1740
Lead	217	25	861

Task. Compare how much energy will be required to cure ~~to~~ 2 different metals.

More complex models

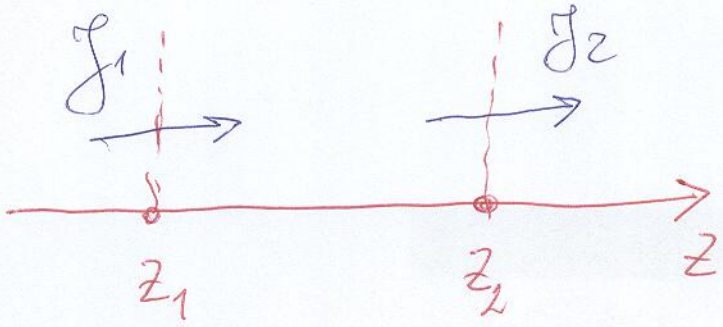
Let consider one important physical process - diffusion of the heat

We assume that the heat flow direction is z coordinate.



It follows from all experiments and our experience that the heat flows from domains (areas) with higher temperetures into colder domains.

~~with~~



$\Delta z = z_2 - z_1$

Our analysis is based on the energy conservation law.

Denote $T = T(t, z)$ the temperature of metal

Initial temperature is $T(0, z)$

During time interval Δt the temperature is changed due to the change of energy amount in this area $[z_1, z_1 + \Delta z]$

$\Delta t \, (J_1 - J_2)$ | J_1 in flow
J_2 out flow.

The temperature is changed from $T(t, z)$ to $T(t+\Delta t, z)$ and the changed of energy can be computed as

$$\rho c \left(T(t+\Delta t, z) - T(t, z) \right) \times \Delta z$$

where

ρ is the density of the material

c - the specific heat capacity of a substance (the amount of heat that must be added to one unit of mass to cause an increase of one unit in temperature.

We get the equation

$$\rho c \left(T(t+\Delta t) - T(t)\right) \Delta z$$

$$= \Delta t \times \left(- J_2 (z+\Delta z) + J(z) \right)$$

$$J_2 = J(z+\Delta z), \quad J_1 = J(z).$$

It follows from results of physical experiments that heat flow is proportional to the gradient of temperature

$$J(z) = - D \frac{\partial T(z)}{\partial z},$$

- (−) states the flow direction is from hot area to cold one
- D is the diffusion coefficient (assume that it is constant)

Substituting this relation into a balance equality we get

$$\rho c \frac{T(t+\Delta t) - T(t)}{\Delta t} = \lambda \frac{\frac{\partial T}{\partial z}(z+\Delta z) - \frac{\partial T}{\partial z}(z)}{\Delta z}$$

Taking the limit $\Delta t, \Delta z \to 0$ we define the mathematic model describing heat conduction

$$\rho c \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial z^2} \qquad (*)\ (1)$$

$$T(z,0) = T_0, \quad 0 \le z \le L. \qquad (2)$$

Task. Define dimension of $[\lambda]$

This equation describes heat diffusion inside the metal (material).

Next we must define equation for the processes on a surface for $s(t)$ (boundary condition for $(*)$).

In addition to differential equation (*) we should define boundary conditions.

At the right side boundary $z=L$ it is sufficient to take equation

(3) $T(L,t) = T_0$ (stationary temperature)

Next we want to define the BC for $z=s(t)$ on the surface of metal piece.

Also we want to get a more accurate position of the front $s=s(t)$ (dynamics of it).

It is clear that it is important step to take into account the loss of energy due to liquid evaporation.

We can remember the simple model ~~be~~ constructed above

$$\rho h \frac{ds}{dt} = \frac{W}{A}, \qquad \text{„} L_v \text{ (nedefinirano)}$$

$$h = \underbrace{h_1 (T - T_0)}_{\text{heating of metal}} + \underbrace{h_2}_{\text{melting (liquid)}} + \underbrace{h_3}_{\text{evaporation (gas form)}}$$

Take a short time step Δt:

$$t \le \tilde{t} < t + \Delta t.$$

We consider a space interval

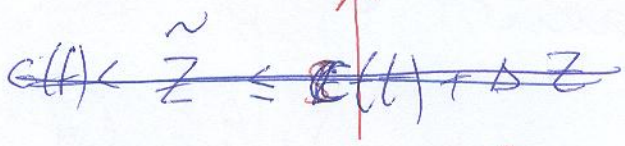
$$s(t) \le \tilde{z} \le s(t + \Delta t),$$

$$\Delta z = s(t + \Delta t) - s(t).$$

The heat (energy) balance is written as:

$$\rho c \left(T(z, t+\Delta t) - T(z,t) \right) \Delta z$$

$$= \left(\mathcal{D} \frac{\partial T}{\partial z}(z+\Delta z, t) + \frac{W}{A} \right) \Delta t$$



laser beam energy

Taking the limit $\Delta z, \Delta t \to 0$ we get the boundary condition for $z = 0$:

$$(4) \quad -\mathcal{D} \frac{\partial T}{\partial z}\bigg|_{z=0} = \frac{W}{A}$$

When temperature reaches the critical value the evaporation process is started. We get the BC

$$T(s(t), t) = T_v$$

(the BC (4) is changed to new one)

$$\rho c \left(T(\tilde{z}, t+\Delta t) - T(\tilde{z}, t) \right) \Delta z$$

$$= \left(\lambda \frac{\partial T}{\partial z}(z+\Delta z, t) + \frac{W}{A} \right) \Delta t$$

$$- \rho L_v \left(s(t+\Delta t) - s(t) \right)$$

Taking the limit $\Delta t, \Delta z \to 0$ we get the equation of front $s(t)$ movement

1° L°

$$\begin{cases} \rho L_v \dfrac{ds}{dt} = \dfrac{W}{A} + \lambda \dfrac{\partial T}{\partial z}(s(t), t) \\ s(t_0) = 0 \end{cases}$$